Theta Individual	heta Individual 2018 MAO National Conv					
1. Consider four distinct vowels and eight distinct consonants, one being <i>b</i> . How many five-letter "words" can be formed containing two different vowels and three different consonants if the word must contain <i>b</i> ? The "word" need not be an actual word.						
A. 126 B. 15120	C. 20160	D. 4032	E. NOTA			
2. The point equidistant from (–4, 3), (A. 8 B. 12		s (<i>a, b</i>). Find the value D. −4				
3. A linear transformation maps the graph of $y = \frac{1}{x}$ to $y = \frac{4x+5}{x+1}$. Find the equation of the image of $y = x^2 - 2x + 3$ under the same translation.						
A. $y=x^2-4x+3$ B. $y=x^2-4x+10$		D. $y = x^2 - 2x + 7$	E. NOTA			
4. The number $4^{14} - 1$ is divisible by 29 but $2^{14} - 1$ is not. What is the remainder when $2^{14} - 1$ is divided by 29?						
A. 27 B. 14	C. 21	D. 16	E. NOTA			
5. How many distinct prime numbers aA. 29B. 15	are in the first 50 n C. 16	ows of Pascal's Trian D. 28	gle? E. NOTA			
6. Simplify $\left(\frac{1+i}{1-i} - \frac{1-i}{1+i}\right)^5$ if $i = \sqrt{-1}$.						
A32 B32 <i>i</i>	C. 0	D. 2	E. NOTA			
7. A parallelogram has diagonals of length 10 and 20. Find the area enclosed by the circle						
inscribed in the parallelogram. A. 12π B. 8π	С. 20π	D. 16π	E. NOTA			
8. The length of a rectangle increases by 40% and the width decreases by 25%. By what percentage does the area increase?						
A. 115 B. 105	C. 120	D. 110	E. NOTA			
9. Let $f(x) = \frac{2x-3}{x+3}$ and $g(x) = \frac{3x-4}{x+2}$. There are integers <i>a</i> , <i>b</i> , <i>c</i> , and <i>d</i> such that $f(g(x)) = \frac{ax+b}{cx+d}$.						
What is the value of $\frac{b}{d}$?						
A. $\frac{2}{3}$ B. $-\frac{1}{13}$	C. –7	D2	E. NOTA			
10. Two of the roots of $x^3 + 6x^2 - 25x + A$. 6 B6	<i>c</i> =0 are 1 and 2. C. −18	What is the value of <i>c</i> D. 18	e? E. NOTA			

- 11. The function $f(x) = ba^{-x} + c$ has horizontal asymptote y = 32, y-intercept 212, and passes
through (2, 112). Find the value of $a \cdot c + b$.A. 238B. 88C. 152D. 302E. NOTA
- 12. The first three terms of an arithmetic progression and a geometric progression (they are different progressions) are, in order, 3x-2, -4x, $-5x^2$. Find the sum of the second terms of all possible sequences.
 - A. $-\frac{36}{5}$ B. $-\frac{48}{5}$ C. $-\frac{164}{5}$ D. $\frac{9}{5}$ E. NOTA
- 13. Find the sum of all real values of k so that the following system of equations is consistent.

A.
$$\frac{5}{14}$$
 B. -3 C. $-\frac{12}{7}$ D. $-\frac{45}{14}$ E. NOTA

- 14. Two perpendicular chords of a circle intersect at point *M*. One chord is 7 units long, divided at *M* into segments of length 3 and 4, while the other chord is divided into segments of length 2 and 6. What is the area enclosed by the circle?
 - A. 14π B. $\frac{61}{4}\pi$ C. $\frac{65}{4}\pi$ D. $\frac{75}{4}\pi$ E. NOTA
- 15. Find the shortest distance between $\left(\frac{9}{2}, 0\right)$ and the curve $y = \sqrt{x-1}$. A. $\frac{\sqrt{13}}{2}$ B. $\sqrt{3}$ C. $\frac{\sqrt{7}}{2}$ D. $\frac{\sqrt{11}}{2}$ E. NOTA
- 16. A big-box store sells cans of eight tennis balls. The balls are stacked one on top of the other in a cylindrical can where the balls are tangent to the can. What fraction of the volume of the can is inside the balls?
 - A. $\frac{2}{3}$ B. $\frac{1}{3}$ C. $\frac{3}{4}$ D. $\frac{3}{8}$ E. NOTA

17. The range of $y = \frac{3x^2 + 4}{2x + 1}$ can be written in interval notation as $\left(-\infty, \frac{a - \sqrt{b}}{c}\right) \cup \left(\frac{a + \sqrt{b}}{c}, \infty\right)$. Find the value of $\frac{a + b}{c}$. A. 30 B. 35 C. 29 D. 27 E. NOTA 18. Find the determinant of the matrix below.

19. Find the element in the second row and third column of A^{-1} if $A = \begin{bmatrix} 3 & -11 & 4 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$.

A. 3 B.
$$\frac{1}{16}$$
 C. $\frac{3}{16}$ D. -5 E. NOTA

20. The third and fifth terms of a harmonic sequence are $\frac{1}{8}$ and $\frac{1}{17}$. Find the first term.A. -1B. $\frac{1}{6}$ C. $\frac{2}{11}$ D. $\frac{3}{14}$ E. NOTA

21. Find the smallest real *x* for which
$$(5^{x} - 7)^{3} + (25^{x} + 1)^{3} = (25^{x} + 5^{x} - 6)^{3}$$
.
A. $\log_{5} 10 - 1$ B. $\log_{5} 7$ C. $\log_{5} \frac{3}{2}$ D. $\log_{5} 6 - 1$ E. NOTA

22. Find the coefficient of
$$x^{50}$$
 in the expansion of the product
 $(1+2x+3x^2+4x^3+...+101x^{100})(1+x+x^2+x^3+...+x^{25}).$
A. 923 B. 125 C. 501 D. 1001 E. NOTA

- 23. A road 40 feet wide passes through a very old semielliptical tunnel that is as wide as the road and $40\sqrt{7}$ feet deep (from entrance to exit). The tunnel is 30 feet high at the point 5 feet from the edge of the road. If the highway department has determined that the tunnel should be condemned and filled with concrete, how much concrete, in ft³, will be needed? A. $3000\pi\sqrt{105}$ B. $12800\pi\sqrt{105}$ C. 48000π D. 42500π E. NOTA
- 24. In how many ways can seven bananas and six oranges be distributed among four children if
each child must receive at least one banana?A. 200B. 1680C. 1890D. 10080E. NOTA

25. A bug walks from the focus *F* of the graph of $x = \sqrt{\frac{1+9y^2}{4}}$ to a point on the graph. From there, it walks to the point $\left(-\frac{\sqrt{13}}{6}, 0\right)$. Find the positive difference between the distances of these two walks.

- A. $\frac{1}{2}$ B. 1 C. $\frac{2}{9}$ D. $\frac{1}{9}$ E. NOTA
- 26. If r > 0 and $(r + r^{-1}) = \sqrt{5}$, what is the value of $r^3 + r^{-3}$? A. 18 B. 10 C. $5\sqrt{5}$ D. $2\sqrt{5}$ E. NOTA

27. The solutions to $3\sqrt{2x^2-5x-3}+2x^2-5x=7$ can be written in the form $x = \frac{a \pm \sqrt{b}}{c}$, where *a*, *b*, and *c* are positive integers. Find the value of a+b+c. A. 50 B. 66 C. 47 D. 57 E. NOTA

28. A class has four boys and three girls. If the students are called randomly to the office once at a time, what is the probability that they go in alternating boy/girl order?

A. 1	B. 1	$C_{-}\frac{2}{-}$	D. <u>1</u>	E. NOTA
7	4	7	35	

29. Radii divide a circle into nine unequal sectors whose central angles each have an integral number of degrees. What is the maximum measure, in degrees, of the central angle of the smallest sector?

- A. 8 B. 36 C. 9 D. 12 E. NOTA
- 30. If f(x)=a+bx, where $a, b \in \mathbb{R}$ and f(f(f(1)))=29 and f(f(f(0)))=2, find the value of a+b.
 - A. 4 B. $\frac{41}{13}$ C. $\frac{19}{6}$ D. $\frac{35}{12}$ E. NOTA